Section 13.6

Quadric Surfaces

To understand sets $S = \{(x, y, z)|R\}$, it is helpful to look at the **traces**: intersections of simple planes with S. A straightforward extension of 2-dimensional curves to 3-dimensional surfaces are **cylinders**: surfaces that are parallel to a given line l.

Definition: Quadric Surfaces
A **quadric surface** is the solution set to the second-degree equation

$$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0$$

The 6 primary quadric surfaces

Example 13.6.1. The surface defined by the equation $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ is an *ellipsoid*.

Example 13.6.2. The surface defined by the equation $z = \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2}$ is an *elliptic paraboloid*.

Example 13.6.3. The surface defined by the equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$ is an *elliptic hyperboloid of one sheet*.

Example 13.6.4. The surface defined by the equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0$ is an *elliptic cone*.

Example 13.6.5. The surface defined by the equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = -1$ is an *elliptic hyperboloid of two sheets*.

Example 13.6.6. The surface defined by the equation $z = \dfrac{x^2}{a^2} - \dfrac{y^2}{b^2}$ is an *hyperbolic paraboloid*.